Cambridge International AS & A Level

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MATHEMATICS **9709/12**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Coefficient of x^2 in $\left(2+\frac{x}{2}\right)^6$ is ${}_6C_2\times 2^4\times(½)^2$ (x^2) (= 60)	**B2,1,0**	3 things wanted –1 each incorrect component, must be multiplied together. Allow ${}_6C_4$, $\binom{6}{4}$ and factorial equivalents. Marks can be awarded for correct term in an expansion.
	Coefficient of x^2 in $(a+x)^5$ is ${}_5C_2\times a^3$ (x^2) $(= 10a^3)$	**B1**	Marks can be awarded for correct term in an expansion.
	$\rightarrow 60 + 10a^3 = 330$	**M1**	Forms an equation '*their 60*' + '*their* $10a^3$' = 330, OK with x^2 in all three terms initially. This can be recovered by a correct answer.
	$a = 3$	**A1**	Condone ±3 as long as +3 is selected.
		5	

Question	Answer	Marks	Guidance
2(i)			A complete method as far as finding a set of values for k by:
	Either $(x-3)^2 + k - 9 > 0$, $k - 9 > 0$		Either completing the square and using '*their* $k - 9$' > or ⩾ 0 OR
	or $2x - 6 = 0 \rightarrow (3, k-9)$, $k - 9 > 0$	**M1**	Differentiating and setting to 0, using '*their* $x=3$' to find y and using '*their* $k - 9$' > or ⩾0 OR
	or $b^2 < 4ac$ oe $\rightarrow 36 < 4k$		Use of discriminant < or ⩽ 0. Beware use of > and incorrect algebra.
	$\rightarrow k > 9$ Note: not ⩾	**A1**	T&I leading to (or no working) correct answer 2/2 otherwise 0/2.
		2	

Question	Answer	Marks	Guidance
2(ii)	**EITHER**		
	$x^2 - 6x + k = 7 - 2x \rightarrow x^2 - 4x + k - 7 \ (= 0)$	*M1	Equates and collects terms.
	Use of $b^2 - 4ac = 0$ $(16 - 4(k - 7) = 0)$	DM1	Correct use of discriminant = 0, involving k from a 3 term quadratic.
	OR		
	$2x - 6 = -2 \rightarrow x = 2$ $(y = 3)$	*M1	Equates their $\frac{dy}{dx}$ to ± 2, finds a value for x.
	(*their* 3) or 7– 2(*their* 2) = (*their* 2)^2 – 6(*their* 2) + k	DM1	Substitutes their value(s) into the appropriate equation.
	$\rightarrow k = 11$	A1	
		3	

Question	Answer	Marks	Guidance
3(i)	$r = 1.02$ or $\frac{102}{100}$ used in a GP in some way.	B1	Can be awarded here for use in S_n formula.
	Amount in 12th week = 8000 (*their r*)^{11} or (*their a from* $\frac{8000}{\text{their } r.}$) (*their r*)^{12}	M1	Use of ar^{n-1} with a = 8000 & $n = 12$ or with a = $\frac{8000}{1.02}$ and $n = 13$.
	= 9950 (kg) awrt	A1	Note: Final answer of either 9943 or 9940 implies M1. Full marks can be awarded for a correct answer from a list of terms.
		3	

Question	Answer	Marks	Guidance
3(ii)	In 12 weeks, total is $\frac{8000\left(\left(their\ r\right)^{12}-1\right)}{\left(\left(their\ r\right)-1\right)}$	**M1**	Use of S_n with a = 8000 and n = 12 or addition of 12 terms.
	= 107000 (kg) awrt	**A1**	Correct answer but no working 2/2
		2	

Question	Answer	Marks	Guidance
4(i)	$a + ½b = 5$	**B1**	Alternatively these marks can be awarded when ½ and –1 appear after a or b has been eliminated.
	$a – b = 11$	**B1**	
	→ $a = 7$ and $b = – 4$	**B1**	
		[3]	
4(ii)	$a + b$ or *their a* + *their b* (3)	**B1**	Not enough to be seen in a table of values – must be selected. Graph from their values can get both marks. **Note: Use of $b^2 – 4ac$ scores 0/3**
	$a – b$ or *their a* – *their b* (11).	**B1**	
	→ $k < 3$, $k > 11$	**B1**	Both inequalities correct. Allow combined statement as long as correct inequalities if taken separately. Both answers correct from T & I or guesswork 3/3 otherwise 0/3
		3	

Question	Answer	Marks	Guidance
5(i)	$\overrightarrow{DA} = 6\mathbf{i} - 4\mathbf{k}$	**B1**	
	$\overrightarrow{CA} = 6\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}$	**B1**	
		2	
5(ii)	Method marks awarded only for ***their*** vectors ± $\overrightarrow{CA}$ & ± $\overrightarrow{DA}$		Full marks can be obtained using $\overrightarrow{AC}$ & $\overrightarrow{AD}$
	$\overrightarrow{CA}.\overrightarrow{DA} = 36 + 16\ (= 52)$	**M1**	Using $x_1x_2+y_1y_2+z_1z_2$
	$\left\|\overrightarrow{DA}\right| = \sqrt{52}$, $\left\|\overrightarrow{CA}\right| = \sqrt{77}$	**M1**	Uses modulus twice
	$52 = \sqrt{77}\sqrt{52}\cos C\hat{A}D$ oe	**M1**	All linked correctly
	Cos $C\hat{A}D = 0.82178.. \rightarrow C\hat{A}D = 34.7^\circ$ or 0.606^c awrt	**A1**	Answer must come from +ve cosine ratio
		4	

Question	Answer	Marks	Guidance
6(i)	AT or $BT = r\tan\theta$ or $OT = \frac{r}{\cos\vartheta}$	**B1**	May be seen on diagram.
	$\frac{1}{2}r^2 2\theta$, & $\frac{1}{2}\times r\times(r\tan\theta$ or $AT)$ or $\frac{1}{2}\times r\times(\frac{r}{\cos\vartheta}$ or $OT)$ $\sin\theta$	**M1**	Both formulae, ($\frac{1}{2}r^2\theta$, $\frac{1}{2}bh$ *or* $\frac{1}{2}ab\sin\theta$), seen with 2θ used when needed.
	$\frac{1}{2}r^2 2\theta = 2\times\frac{1}{2}\times r\times r\tan\theta - \frac{1}{2}r^2 2\theta$ oe $\rightarrow 2\theta = \tan\theta$ **AG**	**A1**	Fully correct working from a correct statement. Note: $\frac{1}{2}r^2 2\theta = \frac{1}{2}\ r^2\tan\theta$ is a valid statement.
		3	

Question	Answer	Marks	Guidance
6(ii)	$\theta = 1.2$ or sector area = 76.8	**B1**	
	Area of kite = 165 awrt	**B1**	
	164.6 – 76.8 = 87.8 awrt	**B1**	awrt 87.8 with little or no working can be awarded 3/3. SC Final answers that round to 88 with little or no working can be awarded 2/3.
		3	

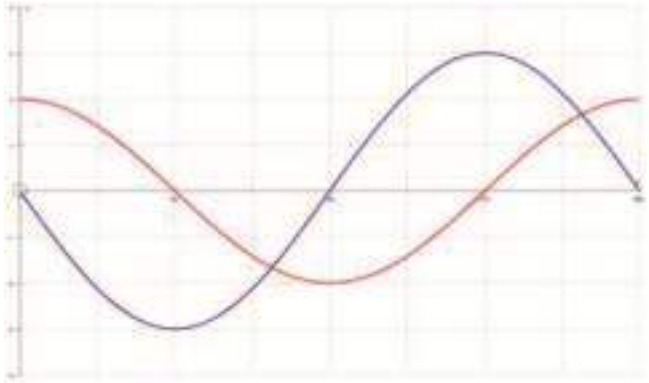
Question	Answer	Marks	Guidance
7(i)	$25 – 2(x + 3)^2$	**B1 B1**	Mark expression if present: B1 for 25 and B1 for $– 2(x + 3)^2$. If no expression award $a = 25$ B1 and $b = 3$ B1.
		2	
7(ii)	(–3, 25)	**B1FT**	FT from answers to (i) or by calculus
		1	
7(iii)	$(k) = –3$ also allow x or $k \geqslant – 3$	**B1FT**	FT from answer to (i) or (ii) **NOT** $x = –3$
		1	

Question	Answer	Marks	Guidance
7(iv)	**EITHER**		
	$y = 25 - 2(x+3)^2 \rightarrow 2(x+3)^2 = 25 - y$	*M1	Makes their squared term containing x the subject or equivalent with x/y interchanged first. Condone errors with +/- signs.
	$x + 3 = (\pm)\sqrt{½(25-y)}$	DM1	Divide by ±2 and then square root allow ±.
	OR		
	$y = 7 - 2x^2 - 12x \rightarrow 2x^2 + 12x + y - 7\ (= 0)$	*M1	Rearranging equation of the curve.
	$x = \dfrac{-12 \pm \sqrt{12^2 - 8(y-7)}}{4}$	DM1	Correct use of their 'a, b and c' in quadratic formula. Allow just + in place of ±.
	$g^{-1}(x) = \sqrt{\left(\dfrac{25 - \boldsymbol{x}}{2}\right)} - 3$ oe isw if substituting $x = -3$	A1	± gets A0. Must now be a function of x. Allow $y =$
		3	

Question	Answer	Marks	Guidance
8	**EITHER**		
	Gradient of bisector $= -\frac{3}{2}$	**B1**	
	gradient $AB = \frac{5h-h}{4h+6-h}$	***M1**	Attempt at $\frac{y-step}{x-step}$
	Either $\frac{5h-h}{4h+6-h} = \frac{2}{3}$ or $-\frac{4h+6-h}{5h-h} = -\frac{3}{2}$	***M1**	Using $m_1m_2 = -1$ appropriately to form an equation.
	OR		
	Gradient of bisector $= -\frac{3}{2}$	**B1**	
	Using gradient of AB and A, B or midpoint $\rightarrow \frac{2}{3}x + \frac{h}{3} = y$ oe	***M1**	Obtain equation of AB using gradient from $m_1m_2 = -1$ and a point.
	Substitute co-ordinates of one of the other points	***M1**	Arrive at an equation in h.
	h = 2	**A1**	
	Midpoint is $\left(\frac{5h+6}{2}, 3h\right)$ or (8, 6)	**B1FT**	Algebraic expression or FT for numerical answer from '*their h*'
	Uses midpoint and '*their h*' with $3x + 2y = k$	**DM1**	Substitutes '*their midpoint*' into $3x + 2y = k$. If $y = -\frac{3}{2}x + c$ is used (expect $c = 18$) the method mark should be withheld until they ×2.
	$\rightarrow k = 36$ soi	**A1**	
		7	

Question	Answer	Marks	Guidance
9(i)	$y = \frac{2}{3}\left(4x+1\right)^{\frac{3}{2}} \div 4 \ (+ \text{C}) \left(= \frac{\left(4x+1\right)^{\frac{3}{2}}}{6} \right)$	**B1 B1**	B1 without ÷ 4. B1 for ÷ 4 oe. Unsimplified OK
	Uses $x = 2$, $y = 5$	**M1**	Uses (2, 5) in an integral (indicated by an increase in power by 1).
	→ $\boldsymbol{c}$ = ½ oe isw	**A1**	No isw if candidate now goes on to produce a straight line equation
		4	
9(ii)	$\frac{\text{d}y}{\text{d}x} = \frac{\text{d}y}{\text{d}t} \div \frac{\text{d}x}{\text{d}t}$		
	$\frac{dx}{dt} = 0.06 \div 3$	**M1**	Ignore notation. Must be 0.06÷3 for M1.
	= 0.02 oe	**A1**	Correct answer with no working scores 2/2
		2	
9(iii)	$\frac{\text{d}^2y}{\text{d}x^2} = ½ \left(4x+1\right)^{-½} \times 4$	**B1**	
	$\frac{\text{d}^2y}{\text{d}x^2} \times \frac{\text{d}y}{\text{d}x} = \frac{2}{\sqrt{4x+1}} \times \sqrt{4x+1} \quad (= 2)$	**B1FT**	Must either show the algebraic product and state that it results in a constant or evaluate it as '= 2'. Must not evaluate at x =2. ft to apply only if $\frac{\text{d}^2y}{\text{d}x^2}$ is of the form $k\left(4x+1\right)^{-½}$
		2	

Question	Answer	Marks	Guidance
10(i)	$2\cos x = -3\sin x \rightarrow \tan x = -\frac{2}{3}$	**M1**	Use of tan=sin/cos to get tan =, or other valid method to find sin or cos =. M0 for $\tan x = +/-\frac{3}{2}$
	$\rightarrow x = 146.3°$ or $326.3°$awrt	**A1 A1FT**	FT for 180 added to an incorrect first answer in the given range. The second A1 is withheld if any further values in the range $0° \leqslant x \leqslant 360°$ are given. Answers in radians score A0, A0.
		3	

Question	Answer	Marks	Guidance
10(ii)	No labels required on either axis. Assume that the diagram is 0º to 360º unless labelled otherwise. Ignore any part of the diagram outside this range.		
		**B1**	Sketch of $y = 2\cos x$. One complete cycle; start and finish at top of curve at roughly the same positive y value and go below the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Sketch of $y = -3\sin x$ One complete cycle; start and finish on the x axis, must be inverted and go below and then above the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Fully correct answer including the sine curve with clearly larger amplitude than cosine curve. Must now be reasonable curves.
			Note: Separate diagrams can score 2/3
		3	
10(iii)	$x < 146.3º$, $x > 326.3º$	**B1FT B1FT**	Does not need to include 0º, 360º. √ from their answers in (i) Allow combined statement as long as correct inequalities if taken separately. SC For two correct values including ft but with $\leqslant$ and $\geqslant$ B1
		2	

Question	Answer	Marks	Guidance
11(i)	$y = \frac{x}{2} + \frac{6}{x} = 4 \rightarrow x = 2$ or 6	**B1 B1**	Inspection or guesswork OK
	$\frac{dy}{dx} = \frac{1}{2} - \frac{6}{x^2}$	**B1**	Unsimplified OK
	When $x = 2$, $m = -1 \rightarrow x + y = 6$ When $x = 6$, $m = \frac{1}{3} \rightarrow y = \frac{1}{3}x + 2$	*__M1__	Correct method for either tangent
	Attempt to solve simultaneous equations	**DM1**	Could solve BOTH equations separately with $y = x$ and get $x = 3$ both times.
	(3,3)	**A1**	Statement about $y = x$ not required.
		6	

Question	Answer	Marks	Guidance
11(ii)	V = $(\pi)\ \int\left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)\ (\mathrm{d}x)$	*M1	Integrate using $\pi\int y^2\mathrm{d}x$ (doesn't need π or dx). Allow incorrect squaring. Not awarded for $\pi\int\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2\ \mathrm{d}x$. Integration indicated by increase in any power by 1.
	Integration → $\frac{x^3}{12}+6x—\frac{36}{x}$	A2,1	3 things wanted —1 each error, allow + C. (Doesn't need π)
	Using limits '*their 2*' to '*their 6*' ($53\frac{1}{3}\pi,\ \frac{160}{3}\pi, 168$ awrt)	DM1	Evidence of their values 6 and 2 from **(i)** substituted into their integrand and then subtracted. $48-\left(-\frac{16}{3}\right)$ is enough.
	Vol for line: integration or cylinder (→ 64π)	M1	Use of πr^2h or integration of 4^2 (could be from $\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2$)
	Subtracts → $10\frac{2}{3}\boldsymbol{\pi}$ oe $\left(\text{e.g.}\frac{32}{3}\pi, 33.5\,\text{awrt}\right)$	A1	

Question	Answer	Marks	Guidance
11(ii)	**OR**		
	V = $(\pi)\ \int 4^2 - \left(\frac{x}{2}+\frac{6}{x}\right)^2\ (\mathrm{d}x)$	**M1 *M1**	Integrate using $\pi\int y^2\mathrm{d}x$ (doesn't need π or dx) Integration indicated by increase in any power by 1.
	$= (\pi)\ \int 16 - \left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)\ (\mathrm{d}x)$		
	$= (\pi)\left[16x - \left(\frac{x^3}{12}+6x-\frac{36}{x}\right)\right]\ (\mathrm{d}x)$	**A2,1**	Or $\left[10x - \frac{x^3}{12}+\frac{36}{x}\right]$
	= **(**π) (48 - 37⅓)	**DM1**	Evidence of their values 6 and 2 from (i) substituted
	$= 10\frac{2}{3}\boldsymbol{\pi}$ oe $\left(\text{eg}\ \frac{32}{3}\pi, 33.5\,\text{awrt}\right)$	**A1**	
		6	